

**Attendance Problems.** Solve each equation.

1.  $3x + 5 = 17$

2.  $r - 3.5 = 8.7$

3.  $4t - 7 = 8t + 3$

4.  $\frac{n+8}{5} = -6$

5.  $2(y - 5) - 20 = 0$

- I can review properties of equality and use them to write algebraic proofs.
- I can identify properties of equality and congruence.

**Vocabulary:** Proof

**Common Core**

**CC.9-12.G.CO.9** Prove theorems about lines and angles.

**CC.9-12.G.CO.10** Prove theorems about triangles.

**CC.9-12.G.CO.11** Prove theorems about parallelograms.

**CC.9-12.G.SRT.4** Prove theorems about triangles.

## 6. What is proof?

**Properties of Equality**

Addition Property of Equality	If $a = b$ , then $a + c = b + c$ .
Subtraction Property of Equality	If $a = b$ , then $a - c = b - c$ .
Multiplication Property of Equality	If $a = b$ , then $ac = bc$ .
Division Property of Equality	If $a = b$ and $c \neq 0$ , then $\frac{a}{c} = \frac{b}{c}$ .
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$ , then $b = a$ .
Transitive Property of Equality	If $a = b$ and $b = c$ , then $a = c$ .
Substitution Property of Equality	If $a = b$ , then $b$ can be substituted for $a$ in any expression.

**Remember!**

The Distributive Property states that

$$a(b + c) = ab + ac.$$

**Video Example 1.** Solve the equation  $-8 = 5n + 2$ . Write a justification for each step.

### 1 Solving an Equation in Algebra

Solve the equation  $-5 = 3n + 1$ . Write a justification for each step.

$$-5 = 3n + 1$$

Given equation

$$\underline{-1} \quad \underline{-1}$$

Subtraction Property of Equality

$$-6 = 3n$$

Simplify.

$$\underline{-6} = \underline{3n}$$

Division Property of Equality

$$-2 = n$$

Simplify.

$$n = -2$$

Symmetric Property of Equality

**Example 1.** Solve the equation  $4m - 8 = -12$ . Write a justification for each step.

**7. Guided Practice:** Solve the equation  $\frac{1}{2}t = -7$ . Write a justification for each step.

**Video Example 2.** To simulate the motion of an object in a computer game, the designer uses the formula  $sr = 3.6p$  to find the number of pixels the object must travel during each second of animation. In the formula  $s$  is the desired speed of the object in kilometers per hour,  $r$  is the scale of pixels per meter, and  $p$  is the number of pixels traveled per second.

The graphics in the game are based on a scale of 8 pixels per meter. The designer wants to simulate a vehicle moving at 54 km/h. How many pixels must the vehicle travel each second. Solve the equation for  $p$  and justify each step.



**2 Problem-Solving Application**

To simulate the motion of an object in a computer game, the designer uses the formula  $sr = 3.6p$  to find the number of pixels the object must travel during each second of animation. In the formula,  $s$  is the desired speed of the object in kilometers per hour,  $r$  is the scale of pixels per meter, and  $p$  is the number of pixels traveled per second.

The graphics in a game are based on a scale of 6 pixels per meter. The designer wants to simulate a vehicle moving at 75 km/h. How many pixels must the vehicle travel each second? Solve the equation for  $p$  and justify each step.

**1 Understand the Problem**

The answer will be the number of pixels traveled per second.

List the important information:

- $sr = 3.6p$
- $s = 75$  km/h
- $p$ : pixels traveled per second
- $r = 6$  pixels per meter

**2 Make a Plan**

Substitute the given information into the formula and solve.

**3 Solve**

$sr = 3.6p$	Given equation
$(75)(6) = 3.6p$	Substitution Property of Equality
$450 = 3.6p$	Simplify.
$\frac{450}{3.6} = \frac{3.6p}{3.6}$	Division Property of Equality
$125 = p$	Simplify.
$p = 125$ pixels	Symmetric Property of Equality

**4 Look Back**

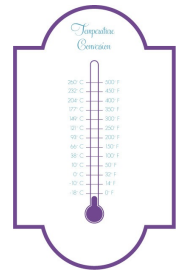
Check your answer by substituting it back into the original formula.

$$\begin{aligned}
 sr &= 3.6p \\
 (75)(6) &= 3.6(125) \\
 450 &= 450 \quad \checkmark
 \end{aligned}$$

**Example 2.** What is the temperature in degrees Fahrenheit  $F$  when it is  $15^{\circ}\text{C}$ ? Solve the equation  $F = \frac{9}{5}C + 32$  for  $F$  and justify each step.

**8. Guided Practice:** What is the temperature in degrees Celsius  $C$  when it is  $86^{\circ}\text{F}$ ? Solve the equation

$$C = \frac{5}{9}(F - 32) \text{ for } C \text{ and justify each step.}$$



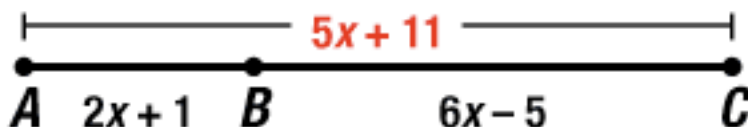
Like algebra, geometry also uses numbers, variables, and operations. For example, segment lengths and angle measures are numbers. So you can use these same properties of equality to write algebraic proofs in geometry.

## Helpful Hint



$AB$  represents the length  $\overline{AB}$ , so you can think of  $AB$  as a variable representing a number.

**Video Example 3.** Write a justification for each step.



Statement	Reason
$AC = AB + BC$	
$5x + 11 = (2x + 1) + (6x - 5)$	
$5x + 11 = 8x - 4$	
$-3x + 11 = -4$	
$-3x = -15$	
$x = 5$	

**3 Solving an Equation in Geometry**

Write a justification for each step.

$$KM = KL + LM$$

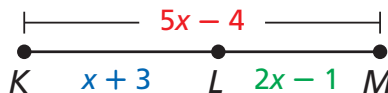
$$5x - 4 = (x + 3) + (2x - 1)$$

$$5x - 4 = 3x + 2$$

$$2x - 4 = 2$$

$$2x = 6$$

$$x = 3$$



Segment Addition Postulate

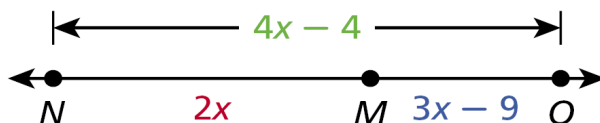
Substitution Property of Equality

Simplify.

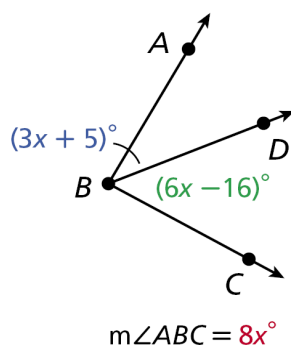
Subtraction Property of Equality

Addition Property of Equality

Division Property of Equality

**Example 3.** Write a justification for each step.

Statement	Reason
$NO = NM + MO$	
$4x - 4 = 2x + (3x - 9)$	
$4x - 4 = 5x - 9$	
$-4 = x - 9$	
$5 = x$	

**9. Guided Practice.** Write a justification for each step.

Statement	Reason
1. $m\angle ABC = m\angle ABD + m\angle DBC$	
2. $8x^\circ = (3x + 5)^\circ + (6x - 16)^\circ$	
3. $8x = 9x - 11$	
4. $-x = -11$	
5. $x = 11$	

You learned in Chapter 1 that segments with equal lengths are congruent and that angles with equal measures are congruent. So the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence.

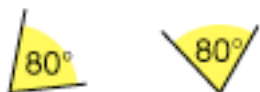
**Properties of Congruence**

SYMBOLS	EXAMPLE
<b>Reflexive Property of Congruence</b> figure $A \cong$ figure $A$ (Reflex. Prop. of $\cong$ )	$\overline{EF} \cong \overline{EF}$
<b>Symmetric Property of Congruence</b> If figure $A \cong$ figure $B$ , then figure $B \cong$ figure $A$ . (Sym. Prop. of $\cong$ )	If $\angle 1 \cong \angle 2$ , then $\angle 2 \cong \angle 1$ .
<b>Transitive Property of Congruence</b> If figure $A \cong$ figure $B$ and figure $B \cong$ figure $C$ , then figure $A \cong$ figure $C$ . (Trans. Prop. of $\cong$ )	If $\overline{PQ} \cong \overline{RS}$ and $\overline{RS} \cong \overline{TU}$ , then $\overline{PQ} \cong \overline{TU}$ .

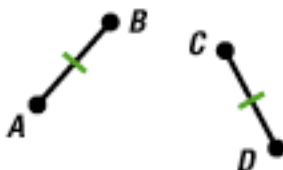
Numbers are equal and shapes are congruent.

**Video Example 4.** Identify the property that justifies each statement.

A.  $80^\circ = 80^\circ$



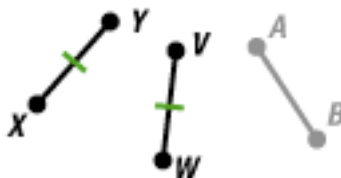
B.  $\overline{AB} \cong \overline{CD}$ , so  $\overline{CD} \cong \overline{AB}$



C.  $\angle G \cong \angle G$



D.  $\overline{XY} \cong \overline{VW}$  and  $\overline{VW} \cong \overline{AB}$ . So  $\overline{XY} \cong \overline{AB}$ .



**4 Identifying Properties of Equality and Congruence**

Identify the property that justifies each statement.

- |          |  |                          |
|----------|--|--------------------------|
| <b>A</b> | $m\angle 1 = m\angle 1$  | Reflex. Prop. of =       |
| <b>B</b> | $\overline{XY} \cong \overline{VW}$ , so $\overline{VW} \cong \overline{XY}$ .             | Sym. Prop. of $\cong$    |
| <b>C</b> | $\angle ABC \cong \angle ABC$  | Reflex. Prop. of $\cong$ |
| <b>D</b> | $\angle 1 \cong \angle 2$ , and $\angle 2 \cong \angle 3$ . So $\angle 1 \cong \angle 3$ . | Trans. Prop. of $\cong$  |

**Example 4.** Identify the property that justifies each statement.

**A.**  $\angle QRS \cong \angle QRS$

**B.**  $m\angle 1 = m\angle 2$  so  $m\angle 2 = m\angle 1$

**C.**  $\overline{AB} \cong \overline{CD}$  and  $\overline{CD} \cong \overline{EF}$ , so  $\overline{AB} \cong \overline{EF}$

**D.**  $32^\circ = 32^\circ$

**Guided Practice:** Identify the property that justifies each statement.

**10.**  $DE = GH$ , so  $GH = DE$ .

**11.**  $94^\circ = 94^\circ$

**12.**  $0 = a$ , and  $a = x$ , So  $0 = x$ .

**13.**  $\angle A \cong \angle Y$  so  $\angle Y \cong \angle A$ .

**2-5 Algebraic Proof:** (p 108) 20-25, 27, 34 & 37.

